Revealed Preferences over Risk and Uncertainty

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Empirical Approaches to Risk and Uncertainty

Models of decision making under risk or under uncertainty often seek to give a better account of observed behavior than expected utility.

Most experimental procedures elicit an agent's preference by collecting a *finite* number of binary choices between risky/uncertain outcomes.

A more recent strand of experimental procedures instead collects a finite number of choices from (typically convex) *budget sets*.

E.g., a subject is presented with a portfolio problem where she has to allocate money between two assets with state-contingent payoffs.

A budgetary choice reveals a preference over an *infinite* number of alternatives; evaluating these data requires a new empirical method.

The GRID Method

The contribution of this paper is to (a) *develop* and (b) *implement* an *empirical method* that can be used to analyze portfolio decisions; it is applicable to budgetary experimental data and to suitable field data.

A *Generalized Restriction of Infinite Domains* (GRID), or the *GRID method*, can be used to test *nonparametrically* the expected utility model and many of its generalizations; it can also be applied to test models of intertemporal choice (such as discounted utility).

Our main methodological result is that a budgetary data set can be rationalized by a given model if (and only if) it can be rationalized on an appropriately modified contingent consumption space $\mathcal{G} \subset \mathbb{R}^S_+$.

- $\mathcal{G}$ is a finite grid that is constructed from the data.

Revealed Preference Analysis

Let $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ be a finite set of price and demand observations which has been collected from an individual consumer.

Every observation consists of a price vector $p^t = (p_1^t, p_2^t, \ldots, p_\ell^t) \gg 0$ and a consumption bundle $x^t = (x_1^t, x_2^t, \ldots, x_\ell^t) \geqslant 0$.

Definition: A utility function $U : \mathbb{R}_+^\ell \to \mathbb{R}$ is said to rationalize the data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ if, at every observation $t = 1, 2, \ldots, T$,

$$U(x^t) \geqslant U(x) \text{ for any } x \in \{x \in \mathbb{R}_+^\ell : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

Afriat (1967) asks the following question: *What are the conditions on $\mathcal{O}$ that are necessary and sufficient for it to have arisen from an agent who is maximizing a nonsatiated utility function?*

Generalized Axiom of Revealed Preference

For any pair (x^t, x^s), we say that x^t is directly revealed preferred to x^s ($x^t \succcurlyeq^* x^s$) if $p^t \cdot x^s \leqslant p^t \cdot x^t$; if $p^t \cdot x^s < p^t \cdot x^t$, then we say that x^t is directly revealed strictly preferred to x^s ($x^t \succ^* x^s$).

Motivation: For an agent maximizing a nonsatiated utility function U,

$$x^t \succcurlyeq^* x^s \implies U(x^t) \geqslant U(x^s),$$

$$x^t \succ^* x^s \implies U(x^t) > U(x^s).$$

Definition: A data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ obeys the Generalized Axiom of Revealed Preference (GARP) if whenever there is a sequence of observations (p^{t_i}, x^{t_i}) (for $i = 1, 2, \ldots, n$) satisfying

$$x^{t_1} \succcurlyeq^* x^{t_2},\, x^{t_2} \succcurlyeq^* x^{t_3},\, \ldots,\, x^{t_{n-1}} \succcurlyeq^* x^{t_n},\, x^{t_n} \succcurlyeq^* x^{t_1},$$

then $\succcurlyeq^*$ cannot be replaced with $\succ^*$ anywhere in the chain.

GARP and Afriat's Theorem

Lemma: A data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ that is collected from an agent who is maximizing a nonsatiated utility function must obey GARP.

Afriat's Theorem: Suppose that $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ satisfies GARP. Then there are real numbers ϕ^t and $\lambda^t > 0$ (for all t) that solve the following system of linear inequalities:

$$\phi^t \leqslant \phi^k + \lambda^k p^k \cdot (x^t - x^k) \text{ for all } k \neq t.$$

Furthermore, $\mathcal{O}$ can be rationalized by $U : \mathbb{R}_+^\ell \to \mathbb{R}$ taking the form

$$U(x) = \min_t \{\phi^t + \lambda^t p^t \cdot (x - x^t)\}.$$

Two things to notice about this result:

(1) Solving linear inequalities is computationally straightforward,

(2) U is increasing, concave, and continuous.

Afriat's Theorem

Afriat's Theorem: The following four statements on the finite set of observations $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ are equivalent:

(1) $\mathcal{O}$ is rationalizable by a nonsatiated utility function U,

(2) $\mathcal{O}$ obeys GARP,

(3) $\mathcal{O}$ satisfies Afriat's inequalities,

(4) $\mathcal{O}$ is rationalizable by a utility function U, which is increasing, concave, and continuous.

Contingent Consumption and Rationalizability

Now suppose that an agent is choosing contingent consumption, i.e.,

$$p^t = (p_1^t, p_2^t, \ldots, p_S^t),$$

$$x^t = (x_1^t, x_2^t, \ldots, x_S^t),$$

are vectors of state prices and contingent consumption, respectively.

The aim of this paper is to develop and implement revealed preference tests on $\mathcal{O}$ for different models of choice under risk and uncertainty.

E.g., if we know the probability of state s to be $\pi_s > 0$, how do we test for rationalizability by expected utility (EU), i.e., that there is a utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^{S} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s u(x_s) \text{ for any } x \in B^t,$$

where $B^t = \{x \in \mathbb{R}_+^S : p^t \cdot x \leqslant p^t \cdot x^t\}$?

Rationalizability by Expected Utility

The standard approach of Varian (1983) and Green and Srivastava (1986) is to *assume* that u is increasing, *concave*, and continuous.

Optimality implies that there is some $\lambda^t > 0$ (for all t) such that

$$\lambda^t p_s^t / \pi_s \in \partial u(x_s^t) \text{ for all } s = 1, 2, \ldots, S, \ \ t = 1, 2, \ldots, T.$$

Therefore, for each (s, t), there is some $\beta_s^t > 0$ such that

$$\frac{\pi_1 \beta_1^t}{p_1^t} = \frac{\pi_2 \beta_2^t}{p_2^t} = \cdots = \frac{\pi_S \beta_S^t}{p_S^t} \text{ for all } t = 1, 2, \ldots, T.$$

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ by an increasing, concave, and continuous utility function u if and only if there is some $\beta_s^t > 0$ (for all (s, t)) such that

(1) whenever $x_s^t > x_{s'}^{t'}$, then $\beta_s^t \leqslant \beta_{s'}^{t'}$,

(2) for every $t = 1, 2, \ldots, T$, $\pi_s \beta_s^t / p_s^t = \pi_{s'} \beta_{s'}^t / p_{s'}^t$. Sufficiency

Rationalizability by Expected Utility

This approach gives a simple linear test, i.e., $\mathcal{O}$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ if and only if there exists a solution to a particular system of linear (in)equalities constructed from $\mathcal{O}$ and π.

But it relies on the *sufficiency* of the first order condition, which holds when the preference over $\mathbb{R}_+^S$ is convex and the budget set is convex.

- Convexity of the preference excludes, e.g., risk loving.
- Convexity of the budget set
  - Excludes nonlinear pricing,
  - Makes it difficult to extend the test in order to measure the 'size' of departures from EU-rationality, which is potentially limiting in many empirical applications.

Our Approach to Testing EU-Rationalizability

Definition: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^{T}$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^{S}$ if there is an increasing and continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every observation $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^{S} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s u(x_s) \text{ for any } x \in B^t.$$

We want to develop a procedure that has the following features:

(1) It tests for EU-rationalizability *as such*, rather than the joint hypothesis of EU-rationalizability *and* global risk aversion,

(2) It can be used to test models of choice other than EU, including those which may allow for nonconvex preferences over contingent consumption, e.g., rank dependent utility,

(3) It is applicable even when budget sets are nonconvex,

(4) It can be adapted to measure the size/significance of departures from a particular model or notion of rationality.

The GRID Test of EU-Rationalizability

Given $\mathcal{O}$, define the set $\mathcal{X} = \{x_s^t : (s,t) \in \{1,\ldots,S\} \times \{1,\ldots,T\}\} \cup 0$, and then the finite grid of points $\mathcal{G} = \mathcal{X}^S$.

E.g., suppose that we observe $x^1 = (2,4)$ chosen from B^1, $x^2 = (6,1)$ chosen from B^2, and $x^3 = (4,3)$ chosen from B^3, with $\pi = (1/2, 1/2)$.

Then, $\mathcal{X} = \{0, 1, 2, 3, 4, 6\}$, and $\mathcal{G} = \mathcal{X} \times \mathcal{X}$.

For EU-rationalizability, it is clearly *necessary* that there are real numbers $\bar{u}(0) < \bar{u}(1) < \cdots < \bar{u}(6)$, such that, at every $t \in \{1,2,3\}$,

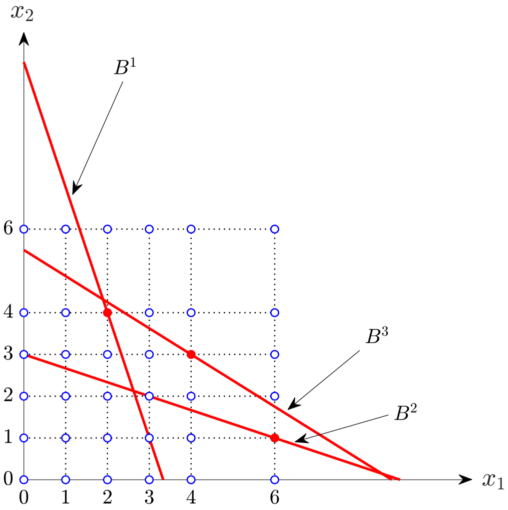
$$\frac{1}{2}\bar{u}(x_1^t) + \frac{1}{2}\bar{u}(x_2^t) \geqslant \frac{1}{2}\bar{u}(x_1) + \frac{1}{2}\bar{u}(x_2) \text{ for any } x \in B^t \cap \mathcal{G},$$

$$\frac{1}{2}\bar{u}(x_1^t) + \frac{1}{2}\bar{u}(x_2^t) > \frac{1}{2}\bar{u}(x_1) + \frac{1}{2}\bar{u}(x_2) \text{ for any } x \in (B^t \backslash \partial B^t) \cap \mathcal{G}.$$

It is also *sufficient* to guarantee EU-rationalizability by an increasing and continuous function $u : \mathbb{R}_+ \to \mathbb{R}$ that extends $\bar{u} : \mathcal{X} \to \mathbb{R}$.

So we only need to check for EU-rationalizability on a finite grid of points, which is a straightforward linear test.

The GRID Test of EU-Rationalizability



$$\mathcal{X} = \{0, 1, 2, 3, 4, 6\}, \quad \mathcal{G} = \mathcal{X} \times \mathcal{X}$$

The GRID Test of EU-Rationalizability

In fact, given the restriction $\bar{u}(0) < \bar{u}(1) < \cdots < \bar{u}(6)$, we only need to check for EU-rationalizability against *undominated* grid points, i.e.,

$$\bar{u}(2) + \bar{u}(4) > \bar{u}(1) + \bar{u}(6), \quad \bar{u}(2) + \bar{u}(4) \geqslant \bar{u}(3) + \bar{u}(1),$$

$$\bar{u}(6) + \bar{u}(1) \geqslant \bar{u}(0) + \bar{u}(3), \quad \bar{u}(6) + \bar{u}(1) \geqslant \bar{u}(3) + \bar{u}(2),$$

$$\bar{u}(4) + \bar{u}(3) > \bar{u}(2) + \bar{u}(4), \quad \bar{u}(4) + \bar{u}(3) > \bar{u}(6) + \bar{u}(1),$$

since $\pi_1 = \pi_2 = 1/2$.

In other words, imposing monotonicity on $\bar{u}$ means that we only need to check against grid points on the *upper boundary* of each budget.

Eliminating redundant EU-rationalizability constraints can in some instances shrink the size of the linear program considerably.

The GRID Test of EU-Rationalizability

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ such that, at every observation $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^{S} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s \bar{u}(x_s) \text{ for any } x \in B^t \cap \mathcal{G},$$

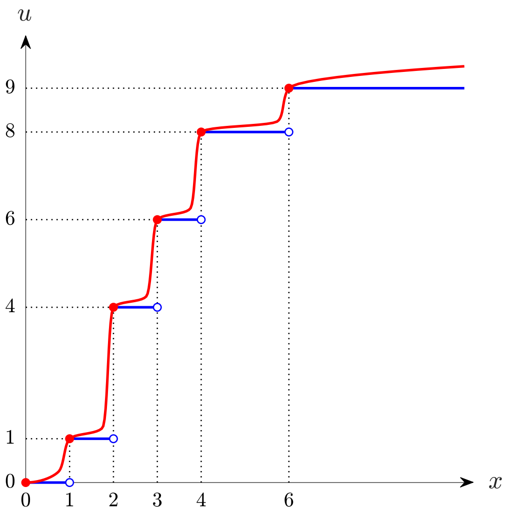
$$\sum_{s=1}^{S} \pi_s \bar{u}(x_s^t) > \sum_{s=1}^{S} \pi_s \bar{u}(x_s) \text{ for any } x \in (B^t \backslash \partial B^t) \cap \mathcal{G}.$$

Intuition: First we replace $\bar{u}$ with the step function $\hat{u} : \mathbb{R}_+ \to \mathbb{R}$ such that $\hat{u}(y) = \bar{u}(y)$ for all $y \in \mathcal{X}$ and $\hat{u}$ is constant between values of $\mathcal{X}$. Clearly, $\hat{u}$ rationalizes the data in the sense that

$$\sum_{s=1}^{S} \pi_s \hat{u}(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s \hat{u}(x_s) \text{ for any } x \in B^t.$$

The only problem is that $\hat{u}$ is neither increasing nor continuous. But it is possible to find another utility function u, arbitrarily close to $\hat{u}$, that is increasing and continuous which also rationalizes the data.

The GRID Test of EU-Rationalizability



$$\mathcal{X} = \{0, 1, 2, \ldots, 6\}, \quad \bar{u}(0) = 0, \quad \bar{u}(1) = 1, \quad \bar{u}(2) = 4, \quad \ldots \ , \quad \bar{u}(6) = 9$$

The GRID Test of EU-Rationalizability

Since the key to our test is the restriction of an infinite consumption space to a finite grid, we refer this new method as the *Generalized Restriction of Infinite Domains* (GRID).

It is 'generalized' because one could think of the domain restriction to a grid as a generalization of the one used in Afriat's Theorem.

- Both involve revealed preference relationships between the chosen bundle x^t and a finite subset of the budget set B^t.
- Afriat makes comparisons with $B^t \cap \mathcal{D}$, where $\mathcal{D} = \{x^t\}_{t=1}^T$.
- The GRID method makes comparisons with $B^t \cap \mathcal{G}$.
- Notice that $\mathcal{G}$ contains $\mathcal{D}$, which occurs since the GRID method characterizes utility with added structure.

The GRID Test in More General Settings

Suppose now that x^t is instead chosen from a compact constraint set $B^t \subset \mathbb{R}^S_+$, so the data set is now $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$.

Typically, the utility function in any particular model of choice under risk or under uncertainty takes the form

$$U(x) = \phi(u(x_1), u(x_2), \ldots, u(x_S)),$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is an increasing and continuous Bernoulli function, and where $\phi : \mathbb{R}^S \to \mathbb{R}$ is an increasing and continuous function drawn from the family Φ, which is specific to the model.

Definition: The data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is ϕ-rationalizable if there is an increasing and continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every observation $t = 1, 2, \ldots, T$,

$$\phi(u(x_1^t), u(x_2^t), \ldots, u(x_S^t)) \geqslant \phi(u(x_1), u(x_2), \ldots, u(x_S))$$

for any $x \in B^t$.

The GRID Test in More General Settings

Given a data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$, define a discrete consumption set $\mathcal{X} = \{x_s^t : (s,t) \in \{1,\ldots,S\} \times \{1,\ldots,T\}\} \cup 0$ and a grid $\mathcal{G} = \mathcal{X}^S$.

Theorem: The data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is ϕ-rationalizable if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ so that, at all $t = 1, 2, \ldots, T$,

$$\phi(\bar{u}(x_1^t), \bar{u}(x_2^t), \ldots, \bar{u}(x_S^t)) \geqslant \phi(\bar{u}(x_1), \bar{u}(x_2), \ldots, \bar{u}(x_S))$$

for any $x \in B^t \cap \mathcal{G}$,

$$\phi(\bar{u}(x_1^t), \bar{u}(x_2^t), \ldots, \bar{u}(x_S^t)) > \phi(\bar{u}(x_1), \bar{u}(x_2), \ldots, \bar{u}(x_S))$$

for all $x \in (B^t \backslash \partial B^t) \cap \mathcal{G}$.

Many models of choice under risk and uncertainty can be described within this framework, with each model leading to a different ϕ.

E.g., expected utility (EU) and subjective expected utility (SEU), rank dependent utility (RDU), disappointment aversion (DA), choice acclimating personal equilibrium (CPE), maxmin expected utility (MEU), and variational preferences (VP). Models

Concave Bernoulli Functions

A common assumption in applications of EU theory is that agents are *risk averse*, which is equivalent to *concavity* of the Bernoulli function.

The GRID method neither requires nor guarantees that the Bernoulli function is concave; this is important because a data set may well be EU-rationalizable, but only with a nonconcave Bernoulli function.

However, we are able to extend the GRID method in order to provide a test for *concave* expected utility (cEU), i.e., EU-rationalizability with a concave Bernoulli function.

The same approach can be applied to test for rank dependent utility with a concave Bernoulli function (cRDU), as well as disappointment aversion with a concave Bernoulli function (cDA), while at the same time continuing to allow for elation seeking.

Implementation on Models of Choice under Risk

We implement an array of tests using data from the portfolio choice experiment in Choi, Fisman, Gale, and Kariv (2007).

93 undergraduate subjects participated in the experiment at UC Berkeley, each completing 50 decision problems under *risk*.

There were two states of the world, each occurring with a *known* probability, and two Arrow-Debreu securities, one for each state.

In each decision problem, every subject was given a budget; income was normalized to one, and state prices were chosen at random.

47 subjects received a symmetric treatment, where $\pi_1 = \pi_2 = 1/2$, and 46 received an asymmetric treatment, where $\pi_1 = 1/3\,(2/3)$.

Implementation on Models of Choice under Risk

Choi *et al.* (2007) first implemented GARP, and then estimated a parametric model of disappointment aversion (Gul, 1991), which contains expected utility as a special case.

We conduct a parallel set of empirical analyses, but we maintain a completely *nonparametric* approach throughout:

- We check GARP in order to test for utility maximization,
- We check F-GARP (Nishimura, Ok, and Quah, 2017) in order to test for *stochastically monotone* utility maximization, i.e., for a utility function obeying *first order stochastic dominance* (FOSD),
- We apply the *GRID method* in order to test (with and without concavity of the Bernoulli function) for
  - Rank dependent utility (RDU/cRDU),
  - Disappointment aversion (DA/cDA),
  - Expected utility (EU/cEU).

Basic Rationalizability Results

$\pi_1 = 1/2$		$\pi_1 \neq 1/2$	
GARP	12/47 (26%)	GARP	4/46 (9%)
F-GARP	1/47 (2%)	F-GARP	3/46 (7%)
RDU	1/47 (2%)	RDU	2/46 (4%)
		DA	1/46 (2%)
EU	1/47 (2%)	EU	1/46 (2%)
cRDU	0/47 (0%)	cRDU	1/46 (2%)
		cDA	1/46 (2%)
cEU	0/47 (0%)	cEU	0/46 (0%)

Table: Pass Rates

The exact pass rates are low across the models we test, which is not surprising given 50 observations on every subject in a rich (in terms of relative price variation) experimental environment.

We need to modify our tests in order to measure the *extent* to which a particular model is able to explain a given data set.

Critical Cost Efficiency Index

In order to accommodate departures from rationality, we adopt an approach first suggested by Afriat (1972, 1973) and Varian (1990).

The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is rationalizable by some family $\mathcal{U}$ if there is a utility function $U : \mathbb{R}^S_+ \to \mathbb{R}$ belonging to $\mathcal{U}$ such that

$$U(x^t) \geqslant U(x) \text{ for any } x \in B^t = \{x \in \mathbb{R}^S_+ : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

If no function in $\mathcal{U}$ rationalizes $\mathcal{O}$, we can make the requirement less stringent by shrinking all budget sets in $\mathcal{O}$ by a factor $e \in [0, 1)$.

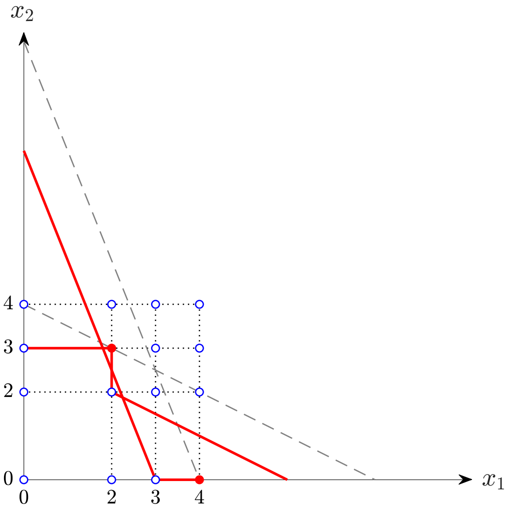
We find U in $\mathcal{U}$ such that $U(x^t) \geqslant U(x)$ for any $x \in B^t(e)$, where

$$B^t(e) = \{x \in \mathbb{R}^S_+ : p^t \cdot x \leqslant e\, p^t \cdot x^t\} \cup \{x \in \mathbb{R}^S_+ : x \leqslant x^t\}.$$

The largest e at which a data set passes the test is known as the critical cost efficiency index (CCEI) associated with $\mathcal{O}$ and $\mathcal{U}$.

Notice that $B^t(e)$ is *not* a convex set, so any approach relying on the sufficiency of first order conditions (see Slide 8) cannot be applied.

Critical Cost Efficiency Index



$$e = 0.75, \ \ (2, 3) \succ_e^* (4, 0), \ \ (4, 0) \not\succ_e^* (2, 3)$$

Rationalizability Results

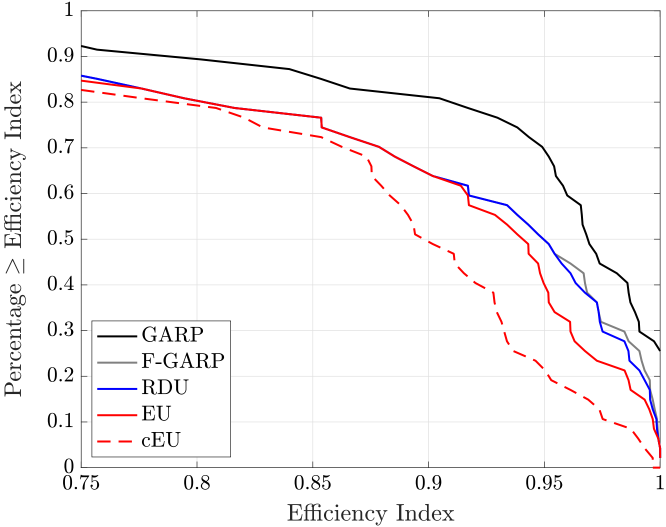
$\pi_1 = 1/2$			$\pi_1 \neq 1/2$		
	$e = 0.90$	$e = 0.95$		$e = 0.90$	$e = 0.95$
GARP	38/47 (81%)	32/47 (68%)	GARP	37/46 (80%)	29/46 (63%)
F-GARP	30/47 (64%)	23/47 (49%)	F-GARP	33/46 (72%)	26/46 (57%)
RDU	30/47 (64%)	23/47 (49%)	RDU	33/46 (72%)	24/46 (52%)
			DA	20/46 (43%)	12/46 (26%)
EU	30/47 (64%)	18/47 (38%)	EU	18/46 (39%)	12/46 (26%)
cRDU	24/47 (51%)	12/47 (26%)	cRDU	25/46 (54%)	14/46 (30%)
			cDA	13/46 (28%)	6/46 (13%)
cEU	23/47 (49%)	10/47 (21%)	cEU	11/46 (24%)	5/46 (11%)

Table: Pass Rates

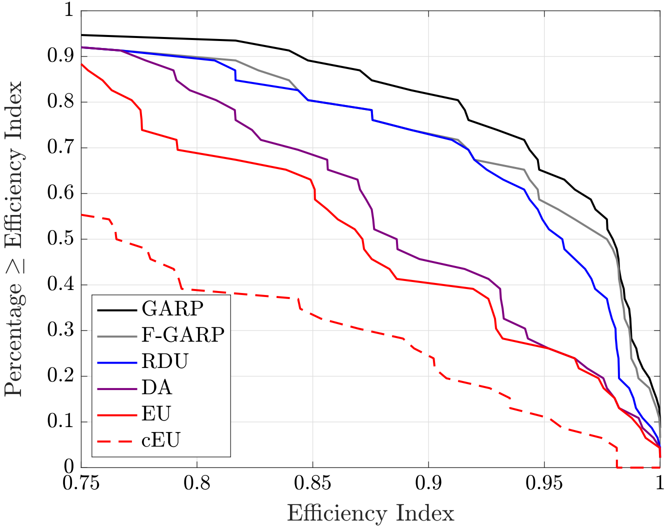
The rationalizability picture changes substantially once we allow for a degree of error in the form of cost inefficiencies.

E.g., about 81% (66%) of subjects pass GARP at efficiency thresholds exceeding 0.9 (0.95), suggesting that a large fraction of subjects does behave in a way that is broadly compatible with utility maximization.

CCEI Distributions ($\pi_1 = 1/2$)



CCEI Distributions ($\pi_1 \neq 1/2$)



Implementation on Models of Choice under Risk

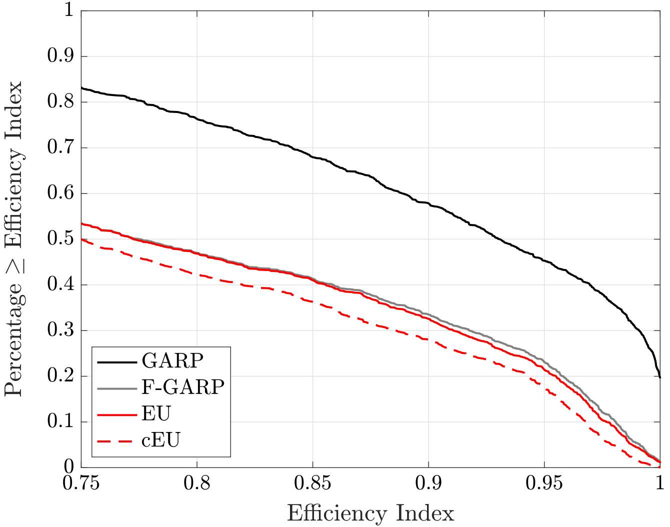
Choi, Kariv, Müller, and Silverman (2014) correlated rationality scores with observable characteristics and other covariates.

- Data were collected on 1,182 subjects in the CentERpanel (NL), each completing 25 portfolio choice problems under symmetric risk; the design was the same as in Choi *et al.* (2007).

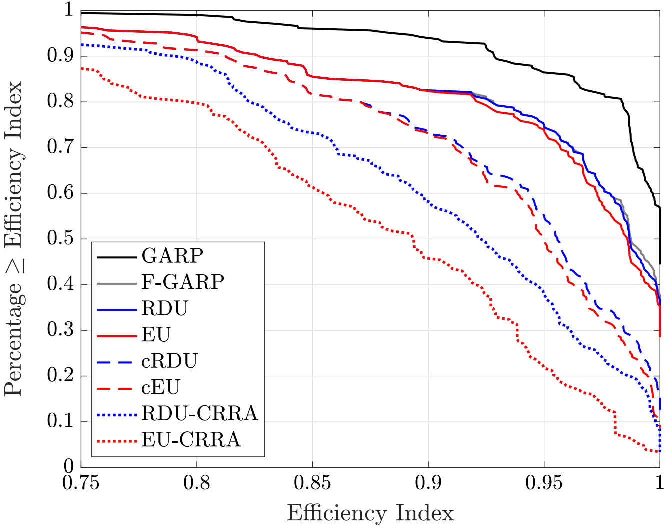
Halevy, Persitz, and Zrill (2018) used a revealed preference approach to decompose the distance to utility maximization into (a) basic inconsistency, and (b) parametric misspecification.

- Data were collected on 207 subjects at UBC, each completing 22 portfolio choice problems; state probabilities were equal, and state prices/income were determined *ex ante*.

CCEI Distributions (Choi *et al.*, 2014)



CCEI Distributions (Halevy, Persitz, and Zrill, 2018)



Summary of Main Empirical Findings

The following highlights some salient features of the data across the three experiments that we examine:

(1) A significant minority of subjects either violate GARP and/or F-GARP; the decisions of these subjects cannot be explained by the EU, DA, or RDU models, all of which respect FOSD.

(2) Around half of the subjects who pass GARP (at some reasonable efficiency threshold) are also compatible with the EU model.

(3) We find little evidence that the DA model accounts for the behavior of subjects not accounted for by the EU model.

(4) There is some evidence that the RDU model explains a significant part of the population not behaving as EU-maximizers.

Conclusions

The GRID approach to testing models of decision making under risk and uncertainty has the following properties:

(1) It avoids ancillary assumptions on the shape of preferences,

(2) It is easy to understand,

(3) It can be easily implemented,

(4) It is flexible enough to measure departures from a model,

(5) It facilitates comparison across models.

Appendix

Rationalizability by Expected Utility

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ by an increasing, concave, and continuous utility function u if and only if there is some $\beta_s^t > 0$ (for all (s,t)) such that

(1) whenever $x_s^t > x_{s'}^{t'}$, then $\beta_s^t \leqslant \beta_{s'}^{t'}$,

(2) for every $t = 1, 2, \ldots, T$, $\pi_s \beta_s^t / p_s^t = \pi_{s'} \beta_{s'}^t / p_{s'}^t$.

Proof of sufficiency: Choose a nonincreasing and continuous function $v : \mathbb{R}_+ \to \mathbb{R}_{++}$ such that $v(x_s^t) = \beta_s^t$ (for all (s,t)). [1]

Define a utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ according to $u(y) = \int_0^y v(z)\, dz$.

Note that the utility function u is increasing, concave, and continuous (since $u'(y) = v(y)$ is positive, nonincreasing, and continuous).

By restriction [2], x^t (for all t) solves the first order conditions for EU-maximization; since u is concave, these conditions are necessary and sufficient to establish a maximum. Back

Testing Models of Choice under Risk/Uncertainty

In principle, the GRID procedure applies to a wide class of decision making problems under risk and under uncertainty.

The conditions for objective expected utility are conveniently *linear*, but for a number of models, our tests are *bilinear*, which is in general a computationally hard problem.

However, many models have special properties that allow for an easy implementation in practice, especially with a small number of states.

Our general solution strategy is to fix any bounded parameters (e.g., a simple probability in the case of two states), and then to solve the corresponding linear problem.

Testing Models of Choice under Risk

In the objective expected utility (EU) model, $\phi(\cdot) = \sum_{s=1}^{S} \pi_s u_s$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) that solves

$$\sum_{s=1}^{S} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s \bar{u}(x_s) \text{ for all } x \in B^t \cap \mathcal{G}, \text{ etc.}$$

In the choice acclimating personal equilibrium (CPE) model (Kőszegi and Rabin, 2007), which contains EU as a special case,

$$\phi(\cdot) = \sum_{s=1}^{S} \pi_s u_s + \frac{1}{2}(1-\lambda) \sum_{s=1}^{S} \sum_{s'=1}^{S} \pi_s \pi_{s'} |u_s - u_{s'}|,$$

where $\lambda \in [0, 2]$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and $\lambda \in [0, 2]$ that solves a set of bilinear inequalities.

This can be implemented straightforwardly by letting λ take different values on $[0, 2]$ and solving the corresponding linear problem.

Testing for Rank Dependent Utility

In the rank dependent utility (RDU) model (Quiggin, 1982), an agent *ranks* contingent claims and *distorts* their cumulative distribution.

An agent maximizing RDU attaches a probability to a state which depends on the relative attractiveness of the outcome in that state.

When there are two states, $\rho_s = g(\pi_s)$ is the distorted value of the true probability π_s (for $s = 1, 2$); if $u_1 \leqslant u_2$, then

$$\phi(u_1, u_2) = \rho_1 u_1 + (1 - \rho_1) u_2,$$

and if $u_1 > u_2$, then

$$\phi(u_1, u_2) = (1 - \rho_2) u_1 + \rho_2 u_2.$$

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and $\{\rho_1, \rho_2\}$; we let ρ_1 and ρ_2 take different values on a fine grid in $[0, 1]^2$, subject to $\rho_1 \leqslant \rho_2$ (if and only if $\pi_1 \leqslant \pi_2$), and perform a series of linear tests.

Testing for Disappointment Aversion

The disappointment aversion (DA) model (Gul, 1991) is a special case of RDU with two states, coinciding when state probabilities are equal.

In the DA model, if $x_H \geqslant x_L$ and the probability of H is π_H, then the agent behaves as though this probability is

$$\gamma(\pi_H) = \frac{\pi_H}{1 + (1 - \pi_H)\beta},$$

for some $\beta \in (-1, \infty)$, and the utility of $(x_H, \pi_H; x_L, 1 - \pi_H)$ is

$$\gamma(\pi_H)u(x_H) + [1 - \gamma(\pi_H)]u(x_L).$$

Gul (1991) classifies $\beta > 0$ as disappointment aversion ($\gamma(\pi_H) < \pi_H$), and $\beta < 0$ as elation seeking; $\beta = 0$ reduces to EU.

For DA, $\phi(u_H, u_L) = \gamma(\pi_H)u_H + [1 - \gamma(\pi_H)]u_L$, and our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and $\beta \in (-1, \infty)$.

Testing Models of Choice under Uncertainty

In the subjective expected utility (SEU) model, $\phi(\cdot) = \sum_{s=1}^{S} \pi_s u_s$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and π_s (for each $s = 1, 2, \ldots, S$) that solves the set of bilinear inequalities

$$\sum_{s=1}^{S} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s \bar{u}(x_s) \text{ for all } x \in B^t \cap \mathcal{G}, \text{ etc.}$$

The maxmin expected utility (MEU) model (Gilboa and Schmeidler, 1989) allows for ambiguity sensitivity; here we need to find a set Π of distributions such that the data can be rationalized according to

$$\phi(\cdot) = \min_{\pi \in \Pi} \left(\sum_{s=1}^{S} \pi_s u_s \right).$$

Again our test involves solving a set of bilinear inequalities; in some cases, this can be simple. Back