Revealed Preferences over Risk and Uncertainty

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Empirical Approaches to Risk and Uncertainty

Models of decision making under risk or under uncertainty often seek to give a better account of observed behavior than expected utility.

Most experimental procedures elicit an agent's preference by collecting a *finite* number of binary choices between risky/uncertain outcomes.

A more recent strand of experimental procedures instead collects a finite number of choices from (typically convex) *budget sets*.

E.g., a subject is presented with a portfolio problem where she has to allocate money between two assets with state-contingent payoffs.

A budgetary choice reveals a preference over an *infinite* number of alternatives; evaluating these data requires a new empirical method.

The Lattice Method

The contribution of this paper is to (a) *develop* and (b) *implement* an *empirical method* that can be used to analyze portfolio decisions; it is applicable to budgetary experimental data and to suitable field data.

The *lattice method* can be used to test *nonparametrically* the expected utility model and many of its generalizations.

Our main methodological result is that a budgetary data set can be rationalized by a given model if (and only if) it can be rationalized on an appropriately modified contingent consumption space $\mathcal{L} \subset \mathbb{R}^S_+$.

- $\mathcal{L}$ is a finite lattice that has been constructed from the data.

Revealed Preference Analysis

Let $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ be a finite set of price and demand observations which have been drawn on an individual consumer.

Every observation consists of a price vector $p^t = (p_1^t, p_2^t, \ldots, p_\ell^t) \gg 0$ and a consumption bundle $x^t = (x_1^t, x_2^t, \ldots, x_\ell^t) \geqslant 0$.

Definition: A utility function $U : \mathbb{R}_+^\ell \to \mathbb{R}$ is said to rationalize the data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ if, at every observation $t = 1, 2, \ldots, T$,

$$U(x^t) \geqslant U(x) \text{ for any } x \in \{x \in \mathbb{R}_+^\ell : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

Afriat (1967) asks the following question: *What are the conditions on $\mathcal{O}$ that are necessary and sufficient for it to have arisen from an agent who is maximizing a nonsatiated utility function?*

Generalized Axiom of Revealed Preference

For any pair (x^t, x^s), we say that x^t is directly revealed preferred to x^s $(x^t \succcurlyeq^* x^s)$ if $p^t \cdot x^s \leqslant p^t \cdot x^t$; if $p^t \cdot x^s < p^t \cdot x^t$, then we say that x^t is directly revealed strictly preferred to x^s $(x^t \succ^* x^s)$.

Motivation: For an agent maximizing a nonsatiated utility function U,

$$x^t \succcurlyeq^* x^s \implies U(x^t) \geqslant U(x^s),$$

$$x^t \succ^* x^s \implies U(x^t) > U(x^s).$$

Definition: A data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^{T}$ obeys the Generalized Axiom of Revealed Preference (GARP) if whenever there is a sequence of observations (p^{t_i}, x^{t_i}) (for $i = 1, 2, \ldots, n$) satisfying

$$x^{t_1} \succcurlyeq^* x^{t_2},\, x^{t_2} \succcurlyeq^* x^{t_3},\, \ldots,\, x^{t_{n-1}} \succcurlyeq^* x^{t_n},\, x^{t_n} \succcurlyeq^* x^{t_1},$$

then $\succcurlyeq^*$ cannot be replaced with $\succ^*$ anywhere in the chain.

GARP and Afriat's Theorem

Lemma: A data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ that is collected from an agent who is maximizing a nonsatiated utility function must obey GARP.

Afriat's Theorem: Suppose that $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ satisfies GARP. Then there are real numbers ϕ^t and $\lambda^t > 0$ (for all t) that solve the following system of linear inequalities:

$$\phi^t \leqslant \phi^k + \lambda^k p^k \cdot (x^t - x^k) \ \text{ for all } \ k \neq t.$$

Furthermore, $\mathcal{O}$ can be rationalized by $U : \mathbb{R}_+^\ell \to \mathbb{R}$ taking the form

$$U(x) = \min_t \{\phi^t + \lambda^t p^t \cdot (x - x^t)\}.$$

Two things to notice about this result:

(1) Solving linear inequalities is computationally straightforward,

(2) U is increasing, concave, and continuous.

Afriat's Theorem

Afriat's Theorem: The following four statements on the finite set of observations $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^{T}$ are equivalent:

(1) $\mathcal{O}$ is rationalizable by a nonsatiated utility function U,

(2) $\mathcal{O}$ obeys GARP,

(3) $\mathcal{O}$ satisfies Afriat's inequalities,

(4) $\mathcal{O}$ is rationalizable by a utility function U, which is increasing, concave, and continuous.

Contingent Consumption and Rationalizability

Now suppose that an agent is choosing contingent consumption, i.e.,

$$p^t = (p_1^t, p_2^t, \ldots, p_S^t),$$

$$x^t = (x_1^t, x_2^t, \ldots, x_S^t),$$

are vectors of state prices and contingent consumption, respectively.

The aim of this paper is to develop and implement revealed preference tests on $\mathcal{O}$ for different models of choice under risk and uncertainty.

E.g., if we know the probability of state s to be $\pi_s > 0$, how do we test for rationalizability by expected utility (EU), i.e., that there is a utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^{S} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s u(x_s) \text{ for any } x \in B^t,$$

where $B^t = \{x \in \mathbb{R}_+^S : p^t \cdot x \leqslant p^t \cdot x^t\}$?

Rationalizability by Expected Utility

The standard approach of Varian (1983) and Green and Srivastava (1986) is to *assume* that u is increasing, *concave*, and continuous.

Optimality implies that there is some $\lambda^t > 0$ (for all t) such that

$$\lambda^t p_s^t / \pi_s \in \partial u(x_s^t) \text{ for all } s = 1, 2, \ldots, S, \ \ t = 1, 2, \ldots, T.$$

Therefore, for each (s, t), there is some $\beta_s^t > 0$ such that

$$\frac{\pi_1 \beta_1^t}{p_1^t} = \frac{\pi_2 \beta_2^t}{p_2^t} = \cdots = \frac{\pi_S \beta_S^t}{p_S^t} \text{ for all } t = 1, 2, \ldots, T.$$

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ by an increasing, concave, and continuous utility function u if and only if there is some $\beta_s^t > 0$ (for all (s, t)) such that

(1) whenever $x_s^t > x_{s'}^{t'}$, then $\beta_s^t \leqslant \beta_{s'}^{t'}$,

(2) for every $t = 1, 2, \ldots, T$, $\pi_s \beta_s^t / p_s^t = \pi_{s'} \beta_{s'}^t / p_{s'}^t$. Sufficiency

Rationalizability by Expected Utility

This approach gives a simple linear test, i.e., $\mathcal{O}$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ if and only if there exists a solution to a particular system of linear (in)equalities constructed from $\mathcal{O}$ and π.

But it relies on the *sufficiency* of the first order condition, which holds when the preference over $\mathbb{R}_+^S$ is convex and the budget set is convex.

- Convexity of the preference excludes, e.g., risk loving.
- Convexity of the budget set
  - Excludes nonlinear pricing,
  - Makes it difficult to extend the test in order to measure the 'size' of departures from EU-rationality, which is potentially limiting in many empirical applications.

Our Approach to Testing EU-Rationalizability

Definition: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ if there is an increasing and continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every observation $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^{S} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s u(x_s) \text{ for any } x \in B^t.$$

We want to develop a procedure that has the following features:

(1) It tests for EU-rationalizability *as such*, rather than the joint hypothesis of EU-rationalizability *and* global risk aversion,

(2) It can be used to test models of choice other than EU, including those which may allow for nonconvex preferences over contingent consumption, e.g., rank dependent utility,

(3) It is applicable even when budget sets are nonconvex,

(4) It can be adapted to measure the size/significance of departures from a particular model or notion of rationality.

The Lattice Test of EU-Rationalizability

Given $\mathcal{O}$, define the set $\mathcal{X} = \{x_s^t : (s,t) \in \{1,\ldots,S\} \times \{1,\ldots,T\}\} \cup 0$, and then the finite lattice $\mathcal{L} = \mathcal{X}^S$.

E.g., suppose that we observe $x^1 = (2,5)$, $p^1 = (5,2)$, $x^2 = (6,1)$, $p^2 = (1,3)$, $x^3 = (4,3)$, $p^3 = (3,4)$, $\pi = (1/2, 1/2)$.

Then, $\mathcal{X} = \{0,1,2,3,4,5,6\}$, and $\mathcal{L} = \mathcal{X} \times \mathcal{X}$.

For EU-rationalizability, it is clearly *necessary* that there are real numbers $\bar{u}(0) < \bar{u}(1) < \cdots < \bar{u}(6)$, such that, at every $t \in \{1,2,3\}$,

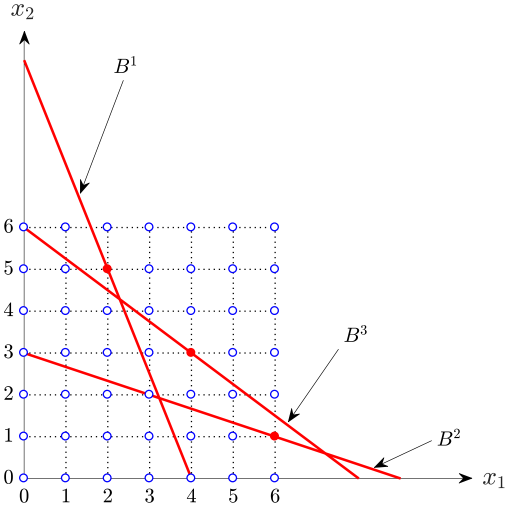
$$\frac{1}{2}\bar{u}(x_1^t) + \frac{1}{2}\bar{u}(x_2^t) \geqslant \frac{1}{2}\bar{u}(x_1) + \frac{1}{2}\bar{u}(x_2) \text{ for any } x \in B^t \cap \mathcal{L},$$

$$\frac{1}{2}\bar{u}(x_1^t) + \frac{1}{2}\bar{u}(x_2^t) > \frac{1}{2}\bar{u}(x_1) + \frac{1}{2}\bar{u}(x_2) \text{ for any } x \in (B^t \backslash \partial B^t) \cap \mathcal{L}.$$

It is also *sufficient* to guarantee EU-rationalizability by an increasing and continuous function $u : \mathbb{R}_+ \to \mathbb{R}$ that extends $\bar{u} : \mathcal{X} \to \mathbb{R}$.

So we only need to check for EU-rationalizability on a finite lattice, which is a straightforward linear test.

The Lattice Test of EU-Rationalizability



$$\mathcal{X} = \{0, 1, 2, 3, 4, 5, 6\}, \quad \mathcal{L} = \mathcal{X} \times \mathcal{X}$$

The Lattice Test of EU-Rationalizability

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ such that, at every observation $t = 1, 2, \ldots, T$,

$$\sum_{s=1}^S \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^S \pi_s \bar{u}(x_s) \text{ for any } x \in B^t \cap \mathcal{L},$$

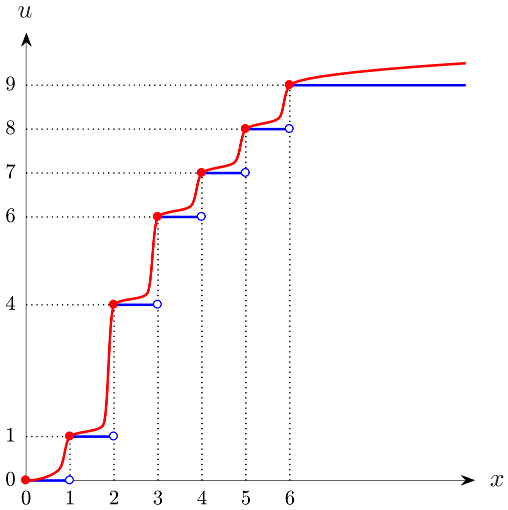
$$\sum_{s=1}^S \pi_s \bar{u}(x_s^t) > \sum_{s=1}^S \pi_s \bar{u}(x_s) \text{ for any } x \in (B^t \backslash \partial B^t) \cap \mathcal{L}.$$

Intuition: First we replace $\bar{u}$ with the step function $\hat{u} : \mathbb{R}_+ \to \mathbb{R}$ such that $\hat{u}(y) = \bar{u}(y)$ for all $y \in \mathcal{X}$ and $\hat{u}$ is constant between values of $\mathcal{X}$. Clearly, $\hat{u}$ rationalizes the data in the sense that

$$\sum_{s=1}^S \pi_s \hat{u}(x_s^t) \geqslant \sum_{s=1}^S \pi_s \hat{u}(x_s) \text{ for any } x \in B^t.$$

The only problem is that $\hat{u}$ is neither increasing nor continuous. But it is possible to find another utility function u, arbitrarily close to $\hat{u}$, that is increasing and continuous which also rationalizes the data.

The Lattice Test of EU-Rationalizability



$$\mathcal{X} = \{0, 1, 2, \ldots, 6\}, \;\; \bar{u}(0) = 0, \;\; \bar{u}(1) = 1, \;\; \bar{u}(2) = 4, \;\; \ldots \;\; , \;\; \bar{u}(6) = 9$$

Independence on the Finite Lattice

The lattice conditions generate a finite list of preference pairs between each x^t and some x in $B^t \cap \mathcal{L}$ or in $(B^t \backslash \partial B^t) \cap \mathcal{L}$.

These pairs form a list $\{(a^j, b^j)\}_{j=1}^M$, where a^j is weakly preferred to b^j for all $j \leqslant N < M$, and a^j is strictly preferred to b^j for all $j > N$.

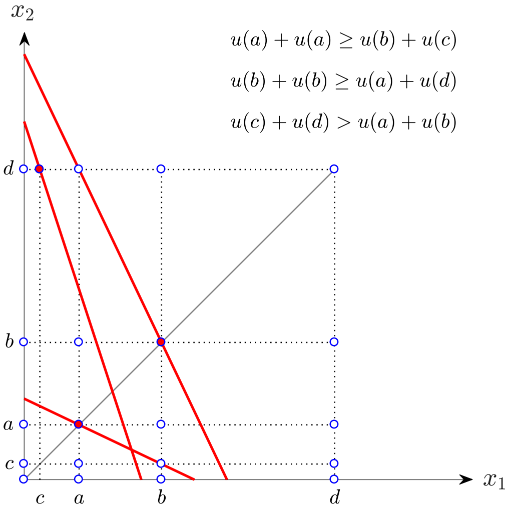
Each bundle a^j (b^j) can be written as the lottery $\hat{a}^j$ ($\hat{b}^j$), where the ith element of $\hat{a}^j$ ($\hat{b}^j$) is the probability of the ith element of $\mathcal{X}$.

From Fishburn (1975), a list $\{(a^j, b^j)\}_{j=1}^M$ is EU-rationalizable if and only if there does not exist λ^j, with $\sum_j \lambda^j = 1$, $\lambda^j \geqslant 0$ for all j, and $\lambda^j > 0$ for some $j > N$, such that

$$\sum_{j=1}^{M} \lambda^j \hat{a}^j = \sum_{j=1}^{M} \lambda^j \hat{b}^j,$$

i.e., independence holds for lotteries induced by the lattice conditions.

A Violation of Expected Utility ($\pi_1 = \pi_2 = 1/2$)



Fishburn's (1975) independence axiom fails; EU-rationalizability holds if and only if Fishburn's axiom holds on the finite lattice $\mathcal{L}$.

The Lattice Test in More General Settings

Suppose now that x^t is instead chosen from a compact constraint set $B^t \subset \mathbb{R}^S_+$, so the data set is now $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$.

Typically, the utility function in particular model of choice under risk or under uncertainty takes the form

$$U(x) = \phi(u(x_1), u(x_2), \ldots, u(x_S)),$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is an increasing and continuous Bernoulli function, and where $\phi : \mathbb{R}^S \to \mathbb{R}$ is an increasing and continuous function drawn from the family Φ, which is specific to the model.

Definition: The data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is ϕ-rationalizable if there is an increasing and continuous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ such that, at every observation $t = 1, 2, \ldots, T$,

$$\phi(u(x^t_1), u(x^t_2), \ldots, u(x^t_S)) \geqslant \phi(u(x_1), u(x_2), \ldots, u(x_S))$$

for any $x \in B^t$.

The Lattice Test in More General Settings

Given the data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$, define the discrete consumption set $\mathcal{X} = \{x_s^t : (s,t) \in \{1,2,\ldots,S\} \times \{1,2,\ldots,T\}\} \cup 0$, and construct the finite lattice $\mathcal{L} = \mathcal{X}^S$.

Theorem: The data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is ϕ-rationalizable if there is an increasing utility function $\bar{u} : \mathcal{X} \to \mathbb{R}$ so that, at all $t = 1, 2, \ldots, T$,

$$\phi(\bar{u}(x_1^t), \bar{u}(x_2^t), \ldots, \bar{u}(x_S^t)) \geqslant \phi(\bar{u}(x_1), \bar{u}(x_2), \ldots, \bar{u}(x_S))$$

for any $x \in B^t \cap \mathcal{L}$,

$$\phi(\bar{u}(x_1^t), \bar{u}(x_2^t), \ldots, \bar{u}(x_S^t)) > \phi(\bar{u}(x_1), \bar{u}(x_2), \ldots, \bar{u}(x_S))$$

for all $x \in (B^t \backslash \partial B^t) \cap \mathcal{L}$.

Many models of choice under risk and uncertainty can be described within this framework, with each model leading to a different ϕ.

E.g., objective and subjective expected utility, rank dependent utility, disappointment aversion, choice acclimating personal equilibrium, maxmin expected utility, and variational preferences. ▸ Models

Implementation on Models of Choice under Risk

We implement an array of tests using data from the portfolio choice experiment in Choi, Fisman, Gale, and Kariv (2007).

93 undergraduate subjects participated in the experiment at UC Berkeley, each completing 50 decision problems under *risk*.

There were two states of the world, each occurring with a *known* probability, and two Arrow-Debreu securities, one for each state.

In each decision problem, every subject was given a budget; income was normalized to one, and state prices were chosen at random.

47 subjects received a symmetric treatment, where $\pi_1 = \pi_2 = 1/2$, and 46 received an asymmetric treatment, where $\pi_1 = 1/3\,(2/3)$.

Implementation on Models of Choice under Risk

Choi *et al.* (2007) first implemented GARP, and then estimated a parametric model of disappointment aversion (Gul, 1991), which contains expected utility as a special case.

We conduct a parallel set of empirical analyses, but we maintain a completely *nonparametric* approach throughout:

- We check GARP in order to test for utility maximization,
- We check F-GARP (Nishimura, Ok, and Quah, 2017) in order to test for *stochastically monotone* utility maximization, i.e., for a utility function which obeys *first order stochastic dominance*,
- We apply the *lattice method* in order to test for
  - Rank dependent utility (Quiggin, 1982),
  - Disappointment aversion (Gul, 1991),
  - Expected utility.

Rationalizability Results

$\pi_1 = 1/2$		$\pi_1 \neq 1/2$	
GARP	12/47 (26%)	GARP	4/46 (9%)
F-GARP	1/47 (2%)	F-GARP	3/46 (7%)
RDU/DA	1/47 (2%)	RDU	2/46 (4%)
		DA	1/46 (2%)
EU	1/47 (2%)	EU	1/46 (2%)

Table: Pass Rates

The exact pass rates are low across the models we test, which is not surprising given 50 observations on every subject in a rich (in terms of relative price variation) experimental environment.

We need to modify our tests in order to measure the *extent* to which a particular model is able to explain a given data set.

Critical Cost Efficiency Index

In order to accommodate departures from rationality, we adopt an approach first suggested by Afriat (1972, 1973) and Varian (1990).

The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^{T}$ is rationalizable by some family $\mathbf{U}$ if there is a utility function $U : \mathbb{R}_+^S \to \mathbb{R}$ belonging to $\mathbf{U}$ such that

$$U(x^t) \geqslant U(x) \text{ for any } x \in B^t = \{x \in \mathbb{R}_+^S : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

If no function in $\mathbf{U}$ rationalizes $\mathcal{O}$, we can make the requirement less stringent by shrinking all budget sets in $\mathcal{O}$ by a factor $e \in [0, 1)$.

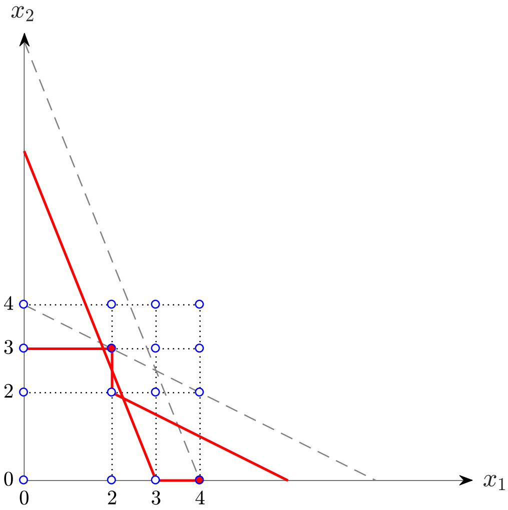
We find U in $\mathbf{U}$ such that $U(x^t) \geqslant U(x)$ for any $x \in B^t(e)$, where

$$B^t(e) = \{x \in \mathbb{R}_+^S : p^t \cdot x \leqslant e\, p^t \cdot x^t\} \cup \{x \in \mathbb{R}_+^S : x \leqslant x^t\}.$$

The largest e at which a data set passes the test is known as the critical cost efficiency index (CCEI) associated with $\mathcal{O}$ and $\mathbf{U}$.

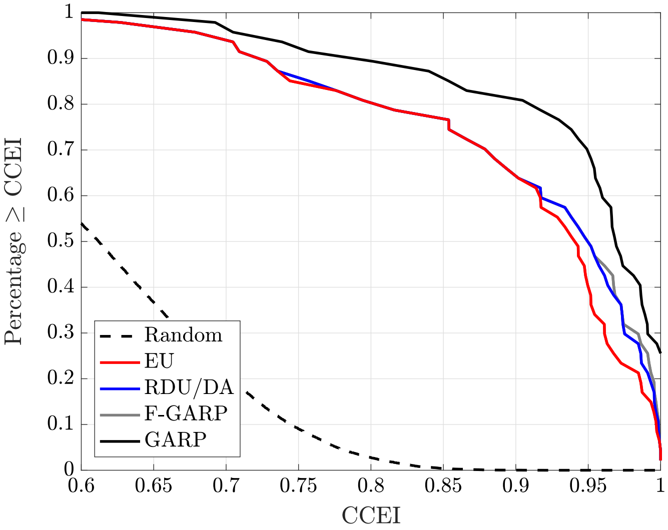
Notice that $B^t(e)$ is *not* a convex set.

Critical Cost Efficiency Index

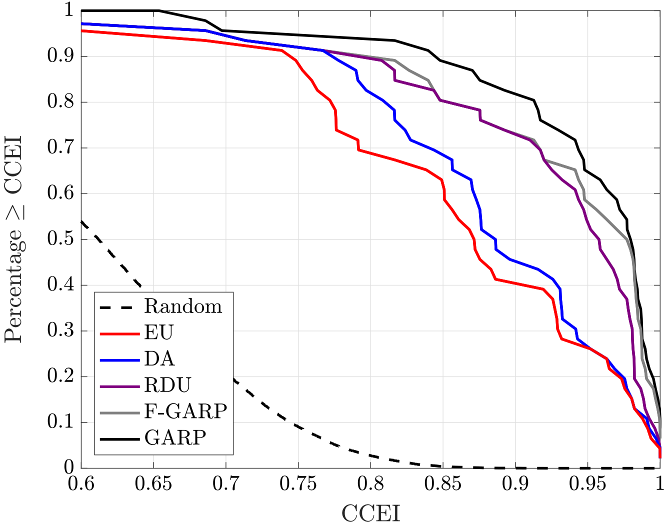


$$e = 0.75, \quad (2,3) \succ_e^* (4,0), \quad (4,0) \not\succeq_e^* (2,3)$$

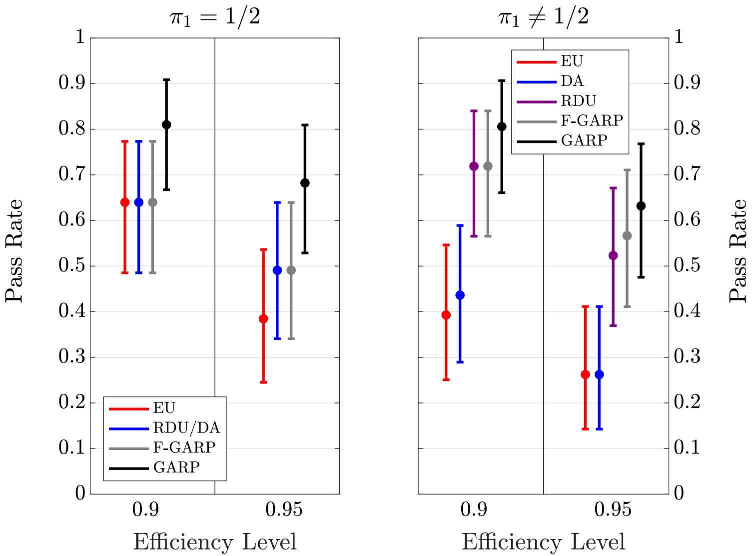
CCEI Distributions ($\pi_1 = 1/2$)



CCEI Distributions ($\pi_1 \neq 1/2$)

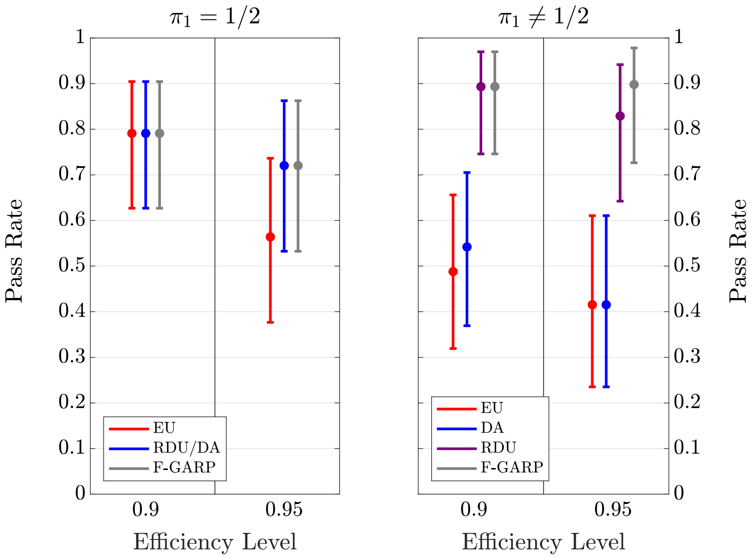


Pass Rates



$\pi_1 = 1/2$
$\pi_1 \neq 1/2$
Pass Rate
Efficiency Level
0.9
0.95
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
EU
RDU/DA
F-GARP
GARP
EU
DA
RDU
F-GARP
GARP

Pass Rates (Conditional on GARP)



Bronars Power

Rationalizability, exact or approximate, only measures whether a given data set is consistent with a particular model; but different models can be more or less observationally stringent.

Bronars (1987) proposed to measure the power of a model/test as the probability of a random (uniform) consumer failing the test.

RDU, DA, and EU are extremely powerful in this environment, i.e., across 50 price-varying budgets, even after conditioning on (F-)GARP.

$\pi_1 = 1/2$			$\pi_1 \neq 1/2$		
	$e = 0.90$	$e = 0.95$		$e = 0.90$	$e = 0.95$
RDU/DA	0.75	0.99	RDU	0.99	1.00
			DA	1.00	1.00
EU	0.87	0.99	EU	1.00	1.00

Table: Power (Conditional on F-GARP)

An Index of Predictive Success

Selten (1991) proposed an index of predictive success to measure the degree to which a model is able to explain a given data set:

$$\text{Hit Rate} - \mu(\text{Model-Consistent Outcomes}).$$

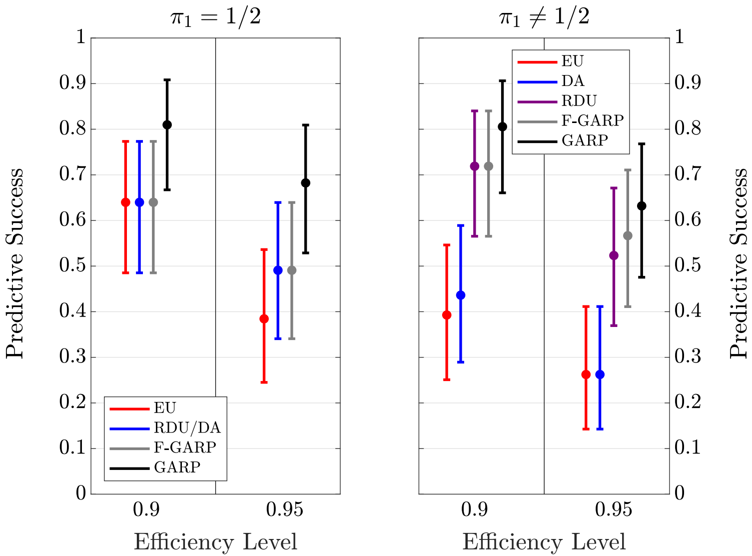
The Hit Rate is the observed frequency of model-consistent outcomes.

The μ(Model-Consistent Outcomes) is the probability of a random outcome being model-consistent; smaller μ means greater *precision*.

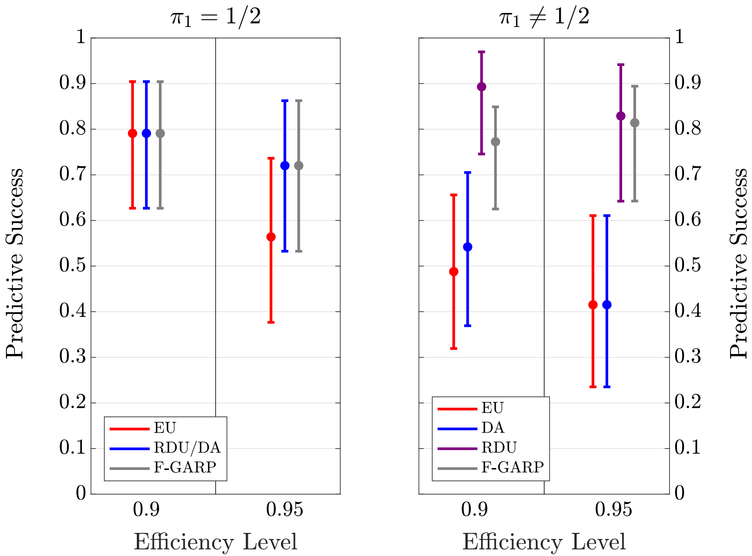
A good model has a high hit rate and high precision (small μ); a poor model has a low hit rate and low precision (large μ).

The Selten index takes values in $[-1, 1]$; any model with a Selten index above 0 can be considered to have some predictive success.

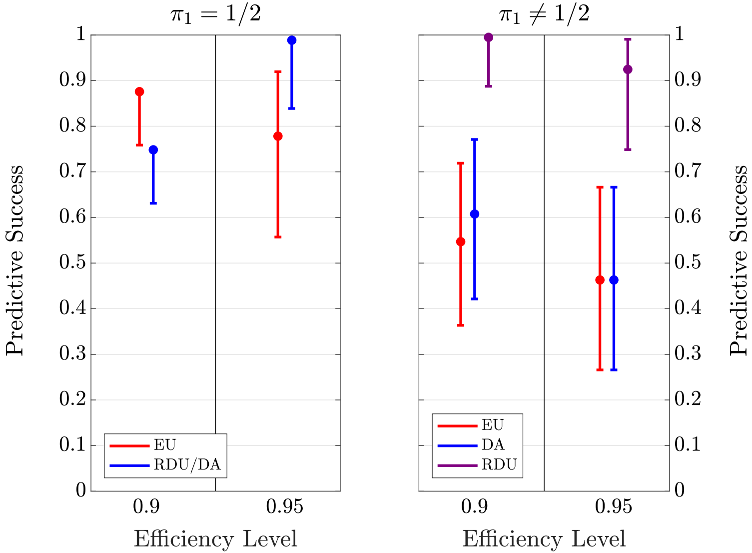
Predictive Success



Predictive Success (Conditional on GARP)



Predictive Success (Conditional on F-GARP)



Summary of Main Empirical Findings

(1) At a cost efficiency threshold of 0.9, more than 80% of subjects are consistent with utility maximization (across treatments).

(2) Nearly 52% of all subjects are rationalizable by EU (64% in the symmetric case, 39% in the asymmetric case).

(3) F-GARP and RDU explain significantly more subjects than EU; almost all subjects who pass F-GARP are rationalizable by RDU.

(4) RDU, DA, and EU are extremely powerful models/tests, even after conditioning on GARP and F-GARP.

(5) All models have predictive success; RDU, DA, and EU continue to have explanatory power even after conditioning on (F-)GARP.

(6) After conditioning on GARP, we find that F-GARP and RDU have the most predictive success; these models explain more behavior than EU, without sacrificing power.

Summary of Main Empirical Findings

Our results appear to be broadly in line with the findings obtained in earlier studies, even though formal comparisons are difficult.

Bruhin, Fehr-Duda, and Epper (2010), Barseghyan *et al.* (2013), etc., also conclude that RDU performs very well.

We find that EU captures a significant proportion of subjects, which is broadly consistent with the fairly common finding that EU puts in a respectable performance (see, e.g., Hey and Orme (1994)).

The 52% pass rate that we report for EU is somewhat higher than in other papers (e.g., 20% in Bruhin, Fehr-Duda, and Epper (2010)).

- Our formulation of EU is about as permissive as it could get.
- We require the Bernoulli function to be increasing in money, but it is estimated nonparametrically and without any curvature assumptions (e.g., concavity).

Conclusions

The lattice approach to testing models of decision making under risk and uncertainty has the following properties:

(1) It avoids ancillary assumptions on the shape of preferences,

(2) It is easy to understand,

(3) It can be easily implemented,

(4) It is flexible enough to measure departures from a model,

(5) It facilitates comparison across models.

Appendix

Rationalizability by Expected Utility

Theorem: The data set $\mathcal{O} = \{(p^t, x^t)\}_{t=1}^T$ is EU-rationalizable with $\pi = \{\pi_s\}_{s=1}^S$ by an increasing, concave, and continuous utility function u if and only if there is some $\beta_s^t > 0$ (for all (s,t)) such that

(1) whenever $x_s^t > x_{s'}^{t'}$, then $\beta_s^t \leqslant \beta_{s'}^{t'}$,

(2) for every $t = 1, 2, \ldots, T$, $\pi_s \beta_s^t / p_s^t = \pi_{s'} \beta_{s'}^t / p_{s'}^t$.

Proof of sufficiency: Choose a non-increasing and continuous function $v : \mathbb{R}_+ \to \mathbb{R}_{++}$ such that $v(x_s^t) = \beta_s^t$ (for all (s,t)). [1]

Define a utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ according to $u(y) = \int_0^y v(z)\, dz$.

Note that the utility function u is increasing, concave, and continuous (since $u'(y) = v(y)$ is positive, non-increasing, and continuous).

By restriction [2], x^t (for all t) solves the first order conditions for EU-maximization; since u is concave, these conditions are necessary and sufficient to establish a maximum. Back

Testing Models of Choice under Risk/Uncertainty

In principle, the lattice procedure applies to a wide class of decision making problems under risk and under uncertainty.

The conditions for objective expected utility are conveniently *linear*, but for a number of models, our tests are *bilinear*, which is in general a computationally hard problem.

However, many models have special properties that allow for an easy implementation in practice, especially with a small number of states.

Our general solution strategy is to fix any bounded parameters (e.g., a simple probability in the case of two states), and then to solve the corresponding linear problem.

Testing Models of Choice under Risk

In the objective expected utility (EU) model, $\phi(\cdot) = \sum_{s=1}^{S} \pi_s u_s$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) that solves

$$\sum_{s=1}^{S} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s \bar{u}(x_s) \text{ for all } x \in B^t \cap \mathcal{L}, \text{ etc.}$$

In the choice acclimating personal equilibrium (CPE) model (Köszegi and Rabin, 2007), which contains EU as a special case,

$$\phi(\cdot) = \sum_{s=1}^{S} \pi_s u_s + \frac{1}{2}(1-\lambda) \sum_{s=1}^{S} \sum_{s'=1}^{S} \pi_s \pi_{s'} |u_s - u_{s'}|,$$

where $\lambda \in [0, 2]$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and $\lambda \in [0, 2]$ that solves a set of bilinear inequalities.

This can be implemented straightforwardly by letting λ take different values on $[0, 2]$ and solving the corresponding linear problem.

Testing for Rank Dependent Utility

In the rank dependent utility (RDU) model (Quiggin, 1982), an agent *ranks* contingent claims and *distorts* their cumulative distribution.

An agent maximizing RDU attaches a probability to a state which depends on the relative attractiveness of the outcome in that state.

When there are two states, $\rho_s = g(\pi_s)$ is the distorted value of the true probability π_s (for $s = 1, 2$); if $u_1 \leqslant u_2$, then

$$\phi(u_1, u_2) = \rho_1 u_1 + (1 - \rho_1) u_2,$$

and if $u_1 > u_2$, then

$$\phi(u_1, u_2) = (1 - \rho_2) u_1 + \rho_2 u_2.$$

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and $\{\rho_1, \rho_2\}$; we let ρ_1 and ρ_2 take different values on a fine grid in $[0, 1]^2$, subject to $\rho_1 \leqslant \rho_2$ (if and only if $\pi_1 \leqslant \pi_2$), and perform a series of linear tests.

Testing for Disappointment Aversion

The disappointment aversion (DA) model (Gul, 1991) is a special case of RDU with two states, coinciding when state probabilities are equal.

In the DA model, if $x_H \geqslant x_L$ and the probability of H is π_H, then the agent behaves as though this probability is

$$\gamma(\pi_H) = \frac{\pi_H}{1 + (1 - \pi_H)\beta},$$

for some $\beta \in (-1, \infty)$, and the utility of $(x_H, \pi_H; x_L, 1 - \pi_H)$ is

$$\gamma(\pi_H)u(x_H) + [1 - \gamma(\pi_H)]u(x_L).$$

Gul (1991) classifies $\beta > 0$ as disappointment aversion ($\gamma(\pi_H) < \pi_H$), and $\beta < 0$ as elation seeking; $\beta = 0$ reduces to EU.

For DA, $\phi(u_H, u_L) = \gamma(\pi_H)u_H + [1 - \gamma(\pi_H)]u_L$, and our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and $\beta \in (-1, \infty)$.

Testing Models of Choice under Uncertainty

In the subjective expected utility (SEU) model, $\phi(\cdot) = \sum_{s=1}^{S} \pi_s u_s$.

Our test involves finding $\bar{u}(y)$ (for each $y \in \mathcal{X}$) and π_s (for each $s = 1, 2, \ldots, S$) that solves the set of bilinear inequalities

$$\sum_{s=1}^{S} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{S} \pi_s \bar{u}(x_s) \text{ for all } x \in B^t \cap \mathcal{L}, \text{ etc.}$$

The maxmin expected utility (MEU) model (Gilboa and Schmeidler, 1989) allows for ambiguity sensitivity; here we need to find a set Π of distributions such that the data can be rationalized according to

$$\phi(\cdot) = \min_{\pi \in \Pi} \left(\sum_{s=1}^{S} \pi_s u_s \right).$$

Again our test involves solving a set of bilinear inequalities; in some cases, this can be simple. Back